No. of Printed Pages : 5

Sem-V-Math-CC-11
(Reg&Back)

2022

Time - 3 hours

Full Marks - 80

*Answer **all groups** as per instructions.*
Figures in the right hand margin indicate marks.
Candidates are required to answer
in their own words as far as practicable.
The symbols used have their usual meanings.

GROUP – A

1. Answer all questions and fill in the blanks as required. [1 × 12

(a) What is removal type of discontinuity ?

(b) $f(x, y) = \tan^{-1}\frac{x}{y}$. Find f_x and f_y.

(c) What is the directional derivative of a scalar field ϕ at a point $P(x, y, z)$ in the direction of **a** ?

(d) What is stationary value of a function $f(x, y)$?

(e) What is the sufficient condition for differentiability ?

(f) $u_1 = f(x, y)$, $u_2 = \phi(x, y)$. Find $\frac{\partial(u_1, u_2)}{\partial(x, y)}$.

(g) Define a vector valued function.

(h) What is grad of a constant function ?

(i) If **a** is vector, find $\nabla(\mathbf{r} \cdot \mathbf{a})$.

(j) $\frac{d\phi}{ds}$ has maximum magnitude along _________ .

(k) State Gauss's Divergence theorem.

(l) State Green's theorem in plane form.

GROUP – B

2. Answer any eight of the following questions. [2 × 8

(a) Find $\lim\limits_{(x,\,y) \to (0,\,0)} xy\left(\frac{x^2 - y^2}{x^2 + y^2}\right)$.

(b) $$f(x, y) = \begin{cases} \dfrac{x^2 - y^2}{x - y} & , (x, y) \neq (0, 0) \\ 0 & , (x, y) = (0, 0). \end{cases}$$

Then find $f_x(0, 0)$.

(c) $u = x^3 + y^4$, $x = t^2$, $y = t^3$. Find $\frac{du}{dt}$.

(d) If $\mathbf{r} = \mathbf{a} \cos wt + \mathbf{b} \sin wt$, find $\mathbf{r} \times \frac{d\mathbf{r}}{dt}$.

(e) Prove that grad $f(r) = f'(r)\frac{\mathbf{r}}{r}$.

(f) Find $\int_0^1 \int_0^1 \frac{x-y}{(x+y)^2}\, dx\, dy$.

(g) Evaluate $\int_C \mathbf{F} \cdot d\mathbf{r}$ where

$\mathbf{F} = 3xy\mathbf{i} - 5z\mathbf{j} + 10x\mathbf{k}$ along the curve C given by

$x = t^2 + 1,\ y = 2t^2,\ z = t^3$ from $t = 1$ to $t = 2$.

(h) Evaluate $\int_0^{\pi} \int_0^{a(1-\cos\theta)} r \sin\theta\, dr\, d\theta$ in the upper half of the cardioide $r = a(1 - \cos\theta)$.

(i) What is the greatest rate of increase of $u = xyz^2$ at (1, 0, 3) ?

(j) $\int_S \mathbf{r} \cdot \mathbf{n}\, ds = 3v$, v is the volume enclosed by 'S'.

GROUP – C

3. Answer any eight questions. [3 × 8

(a) $f(x, y) = (x + y) \sin\frac{1}{x} \sin\frac{1}{y}$, $x > 0$, $y > 0$. Discuss the existence of the repeated limits.

(b) $f(x, y) = 1 \quad , x \neq 0, y \neq 0,$

$\quad\quad\quad = 0 \quad$, when either $x = 0$ or $y = 0$.

Show that the function is discontinuous at (0, 0).

(c) If $u = x^2y$, where $x^2 + xy + y^2 = 1$.

Find $\frac{du}{dx}$.

(d) Show that $x^2 - 2xy + y^2 + x^3 - y^3 + x^5$ has neither maximum nor minimum at origin.

(e) Evaluate $\iint_R (x^2 + y^2)dx\,dy$ where R is the region bounded by $x = 0$, $y = 0$, $x + y = 1$.

(f) Show that the necessary and sufficient condition for a vector function $\mathbf{v}(t)$ to have constant magnitude is $\mathbf{v} \cdot \frac{d\mathbf{v}}{dt} = 0$.

(g) If $\mathbf{F} = (x + 3y)\mathbf{i} + (y - 2z)\mathbf{j} + (x + az)\mathbf{k}$,

find the value of 'a' so that **F** is solenoidal.

(h) Prove that $\text{div}(\mathbf{a} \times \mathbf{r}) = 0$.

(i) Calculate normal derivative at (–1, 1, 1) of

$$f(x, y, z) = yz + zx + xy.$$

(j) Applying Gauss Divergence theorem, show that

$$\int_S \mathbf{F} \times \mathbf{n}\,ds = -\int_V (\text{curl } \mathbf{F})dv$$

GROUP – D

Answer ***any four*** *questions.*

4. Prove that $f(x, y) = |xy|^{\frac{1}{2}}$. Show that f_x and f_y exist at (0, 0) but it is not differentiable at (0, 0). [7

5. Find the equation of the tangent plane and normal line to the surface $yz - zx + xy = -5$ at $(1, -1, 2)$. [7

6. Find the maxima and minima of the function

$$x^3 + y^3 - 63(x + y) + 12xy.$$ [7

7. Prove that curl curl **F** = grad div **F** – Laplacian **F**. [7

8. Evaluate $\iint_R \left(\frac{x^2}{a^2} + \frac{y^2}{b^2}\right) dx\, dy$ over the positive quadrant of the ellipse

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1.$$ [7

9. $\mathbf{F} = 4xz\mathbf{i} - y^2\mathbf{j} + yz\mathbf{k}$. Evaluate $\int_S \mathbf{F} \cdot \mathbf{n}\, ds$ where S is the surface of the cube bounded by $x = 0, x = 1, y = 0, y = 1, z = 0, z = 1$. [7

10. Evaluate $\int_C (xy\, dx + xy^2\, dy)$ by Stoke's theorem, where C is the square in the xy-plane with vertices $(1, 0), (-1, 0), (0, 1)$, and $(0, -1)$. [7

No. of Printed Pages : 5

Sem-V-Math-CC-12
(Reg&Back)

2022

Time - 3 hours

Full Marks - 80

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in their own words as far as practicable.
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GROUP – A

1. Answer all questions and fill in the blanks as required. [1 × 12

(a) Every vector space contains a zero vector. (True / False)

(b) A plane in $\mathbb{R}^3$ not through origin is a subgroup of $\mathbb{R}^3$. Prove it.

(c) What is the span of XY-plane and YZ-plane in $\mathbb{R}^3$. Prove it.

(d) The dimension of $M_{m \times n}(F)$ is ________ .

(e) Is F_3 Bimorphic to $P_3(F)$? Justify your answer.

(f) Define Linear Functional.

(g) The characteristic polynomial of the matrix $A = \begin{pmatrix} 1 & 2 \\ 0 & -1 \end{pmatrix}$ is of degree _______ .

(h) If a linear operator T on an n-dim. vector space V, has n distinct eigen values, then T is _______ .

(i) There exists a linear operator T with no T-invariant subspace. (True / False)

(j) If $A = \begin{pmatrix} 1+i & 3-i \\ -1-i & 2-i \end{pmatrix}$, then $A^{*\prime} =$ _______ .

(k) Write Cuachy-Schwarz Inequality.

(l) Define Real Spectral Theorem.

GROUP – B

2. Answer <u>any eight</u> of the following questions. [2 × 8

(a) For any vector space V, find V/V.

(b) Let $f(t) = e^{rt}$ and $g(t) = e^{st}$, where $r \neq s$. Then prove that the function f and g are L.I.

(c) Let T be the zero transformation. Find N(T) and R(T).

(d) If $f(x) = 3 - 6x + x^2$,

compute $[f(x)]_\beta$, where $\beta = \{1, x, x^2\}$.

(e) $A, B \in M_{n \times n}(F)$.

Show that $tr(AB) = tr(BA)$ and $tr(A) = tr(A^T)$.

(f) Prove that a linear operator T on a finite-dimensional vector space is invertible iff zero is not an eigen value of T.

(g) Prove that $\det(T - \lambda I_v) = \det([T]_\beta - \lambda I)$ for any scalar λ and any ordered basis β for V.

(h) Let V be an inner product space. Then for $x, y \in V$ and $c \in F$. Then prove that $\langle x, cy \rangle = \bar{c} \langle x, y \rangle$.

(i) Write Gram-Schmidt orthogonalization process.

(j) Prove that the matrix $A = \begin{pmatrix} 0 & i \\ i & 0 \end{pmatrix}$ is unitary.

GROUP – C

3. Answer any eight questions. [3 × 8

(a) Let $S = \{x^3 + x^2 + x + 1, x^2 + x + 1, x + 1\}$. Let P be the vector space of polynomial over $\mathbb{R}$. Determine whether the polynomial $-x^3 + 2x^2 + 3x + 3$ is in span of S.

(b) Expand the L.I. subset $S = \{(3, -1, 2)\}$ of $\mathbb{R}^3$ to a basis for $\mathbb{R}^3$.

(c) For subspaces W_1 and W_2,

prove that $W_1 = W_2$ iff $W_1^0 = W_2^0$.

(d) Let $A = \begin{pmatrix} 1 & 3 \\ 4 & 2 \end{pmatrix}$, $V_1 = \begin{pmatrix} 1 \\ -1 \end{pmatrix}$, $V_2 = \begin{pmatrix} 3 \\ 4 \end{pmatrix}$.

Find $[L_A]_\beta$ if $\beta = \{V_1, V_2\}$.

(e) Determine all the eigen values of

$$A = \begin{pmatrix} 2 & 0 & -1 \\ 4 & 1 & -4 \\ 2 & 0 & -1 \end{pmatrix} \text{ for } \mathbb{F} = \mathbb{R}.$$

(f) Every normal operator is not diagnalizable. Prove it.

(g) Find the orthogonal matrix whose first row is $(\frac{1}{3}, \frac{2}{3}, \frac{2}{3})$.

(h) Prove that if $V = W \oplus W^\perp$ and T is the projection on W along $W^\perp$, then $T = T^*$.

(i) T-cyclic subspace of V generated by V is T-invariant. Prove it.

(j) Let V be the vector space of all functions from the field $\mathbb{R}$ into $\mathbb{R}$. If $W = \{f : f(7) = 2 + f(1)\}$, then is W a subspace of V ?

GROUP – D

Answer ***any four*** *questions.*

4. Let V be a vector space and $B = \{v_1, v_2, \ldots\ldots\ldots, v_n\}$ be a subset of V. The B is basis for V iff every $v \in V$ can be uniquely expressed as a linear combination of vectors of B. Proce it. [7

5. Let $T : \mathbb{R}^2 \to \mathbb{R}^3$ be defined by $T(a_1, a_2) = (a_1 - a_2, a_1, 2a_1 + a_2)$. Let β be the standard basis for $\mathbb{R}^2$ and $\gamma = \{(1, 1, 0), (0, 1, 1), (2, 2, 3)\}$. Compute $[T]_\beta^\gamma$. If $\alpha = \{(1, 2), (2, 3)\}$, compute $[T]_\alpha^\gamma$. [7

6. Let V be a finite-dimensional vector space over the field F and let W be a subspace of V. [7

Then prove that $\dim(W) + \dim(W^0) = \dim V$.

7. Let T be a linear operator on a finite-dimensional vector space V and let $\lambda_1, \lambda_2, \lambda_3, \ldots\ldots\ldots\ldots, \lambda_k$ be distinct eigen values of T. If v_i be ab eigen vector of T corresponding to the eigen value λ_i, i = 1, 2, ..., k, then the set $\{v_1, v_2, \ldots\ldots\ldots\ldots, v_k\}$ is linearly dependent. Prove it. [7

8. Let V be an inner product space. Let S = {(1, 1, 0), (1, –1, 1), (–1, 1, 2)}. Let y = (2, 1, 3) ∈ span (S). Show that y is a linear combination of the elements of S. [7

9. Find the minimal solution for the following system of linear equations : [7

$$x + 2y - z = 1$$

$$2x + 3y + z = 2$$

$$4x + 7y - z = 4.$$

Sem-V-Math-DSE-1
(Reg&Back)

2022

Time - 3 hours

Full Marks - 80

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GROUP – A

1. Answer all questions and fill in the blanks as required. [1 × 12

(a) Define Basic feasible solution.

(b) What is a slack variable ?

(c) How to find out the incoming vector ?

(d) If any of the constraints in the primal is a perfect equality, the corresponding dual variable is ________ in sign.

(e) Dual of the dual of a given primal is ________ .

(f) The necessary and sufficient condition for any LPP and its dual to have optimal solution is ________ .

(g) If all $d_{ij} > 0$ in a transportation problem, then the solution under test is ________ .

Sem-V-Math-DSE-1
(Reg&Back)

2022

Time - 3 hours

Full Marks - 80

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GROUP – A

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(b) What is a slack variable ?

(c) How to find out the incoming vector ?

(d) If any of the constraints in the primal is a perfect equality, the corresponding dual variable is ________ in sign.

(e) Dual of the dual of a given primal is ________ .

(f) The necessary and sufficient condition for any LPP and its dual to have optimal solution is ________ .

(g) If all $d_{ij} > 0$ in a transportation problem, then the solution under test is ________ .

(h) What is an unbalanced transportation problem ?

(i) Define a Zero-sum Game.

(j) What is meant by strategy ?

(k) Define a Saddle point.

(l) What is Dominance property ?

GROUP – B

2. Answer any eight of the following questions. [2 × 8

(a) Mark the feasible region represented by constraints

$$x_1 + x_2 \leq 1$$

$$3x_1 + x_2 \geq 3$$

$$x_1 \geq 0 , x_2 \geq 0.$$

(b) State the Fundamental theorem of Linear Programming.

(c) What are two phases of "Two Phase" method ?

(d) State when an LP problem is said to be in standard primal form.

(e) If the primal problem has an unbounded solution, then prove that the dual has either no solution or an unbound solution.

(f) Mention two differences between transportation and an assignment problem.

(g) What is degeneracy in transportation problems ?

(h) Define Assignment problem.

(i) Write two properties of competitive games.

(j) What are Rectangular games ?

GROUP – C

3. Answer any eight questions. [3 × 8

(a) Solve graphically the following LP problem :

Maximise $Z = 3x_1 + 5x_2$

subject to $x_1 + 2x_2 \geq 2$

$3x_1 + x_2 \geq 3$

$4x_1 + 3x_2 \leq 6$

$x_1, x_2 \geq 0$

(b) Solve the following LPP by Simplex method :

Maximise $Z = 2x_1 + x_2$

subject to $x_1 - x_2 \leq 10$

$2x_1 - x_2 \leq 40$

$x_1, x_2 \geq 0$

(c) Find the inverse of the following matrix using Simplex method :

$$A = \begin{bmatrix} 4 & 3 \\ 3 & 2 \end{bmatrix}.$$

(d) Write the dual of the problem :

$$\begin{aligned} \text{Minimise } Z &= 3x_1 + x_2 \\ \text{subject to } \quad & 2x_1 + 3x_2 \geq 2 \\ & x_1 + x_2 \geq 1 \\ & x_1, x_2 \geq 0 \end{aligned}$$

(e) Solve the following minimal assignment problem

Man → Job ↓	1	2	3	4
I	12	30	21	15
II	18	33	9	31
III	44	25	24	21
IV	23	30	28	14

(f) Write the important properties of optimal mixed strategies.

(g) Prove the reduction theorem for an assignment problem.

(h) What is the procedure to get the dual of a mixed system.

(i) Solve the following transportation problem by North-West Corner rule :

To

From		W_1	W_2	W_3	Supply
	F_1	(2)	(7)	(4)	5
From	F_2	(3)	(3)	(1)	8
	F_3	(5)	(4)	(7)	7
	F_4	(1)	(6)	(2)	14
Demand		7	9	18	34

(j) Solve the following game :

B

$$A \quad \begin{array}{c|ccc} & \text{I} & \text{II} & \text{III} \\ \hline \text{I} & 6 & 8 & 6 \\ \text{II} & 4 & 12 & 2 \end{array}$$

GROUP – D

*Answer **any four** questions.*

4. If $x_1 = 2, x_2 = 3, x_3 = 1$ be a feasible solution of the LPP [7

Maximise $Z = x_1 + 2x_2 + 4x_3$

subject to $2x_1 + x_2 + 4x_3 = 11$

[illegible]

and $x_1, x_2, x_3 \ge 0$

[illegible]

[illegible] Solve the following problem by Big-M method : 17

Maximize $Z = x_1 + 2x_2 + 3x_3 - x_4$

subject to $x_1 + 2x_2 + 3x_3 = 15$

$2x_1 + x_2 + 5x_3 = 20$

$x_1 + 2x_2 + x_3 + x_4 = 10$

and $x_1, x_2, x_3, x_4 \ge 0$

[illegible] State and prove Fundamental Duality theorem. 17

[illegible] Prove that if any of the constraints in the primal is a perfect equality, the corresponding dual variable is unrestricted in sign. 17

[illegible] Solve the following transportation problem :

		To			
		1	2	3	Supply
From	1	2	7	4	5
	2	3	3	1	8
	3	5	4	7	7
	4	1	6	2	14
	Demand	7	9	18	34

9. Solve the assignment problem represented by the following matrix : [7

	I	II	III	IV	V	VI
A	9	22	58	11	19	27
B	43	78	72	50	63	48
C	41	28	91	37	45	33
D	74	42	27	49	39	32
E	36	11	57	22	25	18
F	3	56	53	31	17	28

10. Solve the following game graphically : [7

$$\begin{array}{cc|cc|} & & \multicolumn{2}{c}{B} \\ & & \mathrm{I}(y_1) & \mathrm{II}(y_2) \\ \cline{3-4} & \mathrm{I} & 2 & 7 \\ A & \mathrm{II} & 3 & 5 \\ & \mathrm{III} & 11 & 2 \\ \cline{3-4} \end{array}$$

No. of Printed Pages : 5

Sem-V-Math-DSE-2
(Reg&Back)

2022

Time - 3 hours

Full Marks - 80

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GROUP – A

1. Fill in the blanks (all). [1 × 12

(a) Maximum value of probability is ________ .

(b) The function $f(x) = kx$ in $0 < x < 1$ is a valid probability density function if $k =$ ________ .

(c) If $f(x) = Ax^2$ in $0 \leq x \leq 1$ is a probability function, then $A =$ ________ .

(d) If X_1 and X_2 are random variables and a, b are constants, then $E(aX_1 + bX_2) =$ ________ .

(e) If X is uniformly distributed in [a, b], then $E(X) =$ ________ .

(f) The probability of getting 2 heads in tossing of 5 coins is ________ .

P.T.O.

(g) The mean of binomial distribution is ______ .

(h) The mean and variance of Poisson distribution are ______ .

(i) If P(1) = P(2), then the mean of Poisson distribution is ______ .

(j) Highest range of coefficient of correlation is ______ .

(k) The area under the whole normal curve is ______ .

(l) The distribution in which mean, median and mode are equal, is ______ .

GROUP – B

2. Answer <u>any eight</u> of the following questions. [2 × 8

(a) Prove that the probability of the sample space is 1.

(b) If density function $f(x) = \frac{2x}{9}$, $0 \leq x \leq 3$, find mean.

(c) If a coin is tossed 3 times, find probability of obtaining 2 heads.

(d) For any two events A and B, prove that

$$P(AB) \geq P(A) + P(B) - 1.$$

(e) State Chebyshev inequality.

(f) State Central Limit theorem.

(g) Define moment generating function of a binomial distribution.

(h) If X denotes the number of heads in a single toss of 4 fair coins, find $P(X < 2)$.

(i) Show that variance of random variable X is

$$\sigma^2 = E(X^2) - \left(E(X)\right)^2.$$

(j) Test whether the equations $2x + 3y = 4$ and $x + y = 5$ represent valid regression lines.

GROUP – C

3. Answer any eight questions. [3 × 8

(a) For any two events A and $B \subseteq S$ and $A \subseteq B$, prove that

$$P(A) \le P(B) \text{ and } P(B - A) = P(B) - P(A).$$

(b) If A and B are independent events, then prove that A^C, B^C are independent.

(c) Let $f(x, y) = \begin{cases} x + y \ , & 0 \le x \le 1, 0 \le y \le 1 \\ 0 \ , & \text{otherwise.} \end{cases}$

Find conditional density function of X, given Y.

(d) For any random variable X and any constant C, prove that $E(C) = C$.

(e) Let X be the outcome when a fair die is thrown. Find moment generating function.

(f) Two random variables X and Y are defined as $Y = 4X + 9$. Find correlation coefficient between X and Y.

(g) Find binomial distribution whose mean is 5 and variance is $\frac{10}{3}$.

(h) If a Poisson distribution is such that $P(X = 1) \cdot \frac{3}{2} = P(X = 3)$, find $P(X \geq 1)$.

(i) Find the angle between two regression lines.

(j) Let random variables X and Y have joint density function given by

$$f(x) = \begin{cases} c(2x + y) & , 2 < x < 6, 0 < y < 5 \\ 0 & , \text{otherwise.} \end{cases}$$

Find value of c.

GROUP – D

*Answer **any four** questions.*

4. Two dice are thrown. Let A be the event that the sum of points on the face is 9. Let B be the event that at least one number is 6. Find $P(A^C \cup B^C)$. [7

5. A continuous random variable has the probability density function [7

$$f(x) = \begin{cases} kxe^{-\lambda x} & \text{for } x \geq 0, \lambda > 0 \\ 0 & \text{, otherwise.} \end{cases}$$

Find k, mean, variance.

6. Find the probability of getting an even number 3 or 4 or 5 times in throwing 10 dice using binomial distribution. [7

7. Explain moment generating function of Poisson distribution. [7

8. State and prove Central Limit theorem. [7

9. Explain mean, variance of hypergeometric distribution. [7

10. If $E(X^2)$ is finite, then so is $E(|X|)$ and $\{E(|X|)\}^2 \leq E(X^2)$. [7